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DOI:10.1016/j.jedc.2009.03.004

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Life-Cycle Portfolio Choice: The Role of Heterogeneous Under-Diversification*

Claudio Campanale

November 10, 2008

*I wish to thank Carl Chiarella (the editor), three anonymous referees, Rui Albuquerque, Fatih Guvenen, Michael Haliassos, Per Krusell, Giovanna Nicodano and seminar participants at CeRP, the II REDg Workshop 2007, and the 2007 CeRP Workshop on the Economics of Aging for helpful comments. Financial support from the Ministerio de Educación y Ciencia proyecto SEJ 2007-62656, CeRP and IVIE is gratefully acknowledged. Any remaining errors or inconsistencies are entirely my responsibility.

Abstract

In life-cycle portfolio choice models it is standard to assume that all agents invest in a diversified stock market index. In contrast recent empirical evidence, summarized in Campbell (2006) suggests that households' financial portfolios are under-diversified and that there is substantial heterogeneity in diversification. In the present paper I examine the effects of heterogeneous under-diversification in a life-cycle portfolio choice model with uninsurable uncertain earnings and fixed per period participation costs. The analysis of the model shows that realistically calibrated under-diversification gives an important contribution to the explanation of two key facts of households' portfolio allocation: the moderate stock market participation rate and the moderate stock share for participants.

Keywords: Portfolio choice, life-cycle, under-diversification.

JEL codes: G11, D91, H55

1 Introduction

Traditional life-cycle portfolio choice models with intermediate consumption and uninsurable labor income have typically explored investors' decisions about how to allocate wealth between a risk-free and a risky asset. The assumption common to these models is that all agents face the same risky asset that can be interpreted as a stock index fund. This assumption is contradicted by abundant empirical evidence that documents that households invest in a limited number of individual stocks or mutual funds thus facing substantial idiosyncratic risk on their equity investment.[1] The empirical evidence also suggests that more financially sophisticated households, defined by greater education and wealth hold better diversified equity portfolios. In the present paper I explore the effects of portfolio under-diversification on household life-cycle asset allocation. I find that this so far overlooked feature of households' investment strategy substantially improves the ability of the model to rationalize two key empirical facts: the low stock market participation rate in the population and the moderate stock share for market participants.

The model presented here is standard in most respects. It is characterized by finitely lived households that go through the stages of working life and retirement. During working life they face idiosyncratic earnings uncertainty around a deterministic hump-shaped trend. In retirement they face constant and progressive social security benefits. They solve an optimal consumption-saving problem and make an asset allocation decision from a menu of financial assets. Asset demand is subject to a borrowing and short sale constraint. Payment of a one

[1] See for example Curcuru et al. (2007) and Polkovnichenko (2005) for the US and Calvet et al. (2007 and 2008) for Sweden.

time initial entry cost is needed to gain access to the risky asset market.[2] The key departure from the traditional framework is the assumption that there are two mutually exclusive risky financial assets with the same mean but different standard deviation of returns. The two assets are meant to capture in a stylized way the idea of a well diversified and a poorly diversified stock portfolio. On top of the initial entry cost, investors must pay a fixed cost in each period in which they want to participate in the stock market and this cost is higher for the risky asset with lower standard deviation of returns.

The main result of this research is that allowing for under-diversification of households' stock portfolios provides an explanation to two key empirical observations: the low stock market participation rate and the moderate portfolio stock share for participants. [3] The intuition for this result is the following: the increased volatility of the low cost risky asset implies that the optimal share conditional on investing in it is lower. Agents with low wealth do not find it optimal to pay the larger cost needed to buy the well diversified stock portfolio and buy the poorly diversified one, thus lowering the average conditional share in the population. At the same time both the reduced optimal share and the increased variance of returns decrease the benefits of participation for these agents. This, in the presence of the initial entry cost, deters part of them from participating altogether, thus helping to reduce the average participation rate as

[2]The core of these assumptions is shared by most other models in the literature. See for example Campbell et al. (2001), Cocco et al. (2005) and Gomes and Michaelides (2005) for finite-horizon models; Haliassos and Michaelides (2003) and Heaton and Lucas (1997 and 2000) for infinite horizon models.

[3]There is by now a large literature documenting this and other empirical facts about households' portfolio allocation. Two very useful surveys that summarize those findings are Curcuru et al. (2007) and the book edited by Guiso et al. (2001).

well. The interesting finding of the quantitative analysis of the model is that the amount of heterogeneity in the volatility of individual stock portfolios needed to rationalize participation rates and conditional stock shares falls well within the available empirical evidence.

Previous attempts at rationalizing low participation rates and conditional shares had focused on background risk and/or risk aversion. As it is well explained in Gomes and Michaelides (2005) this line of attack carries an implicit tension: increasing risk aversion or background risk reduces the portfolio share of stock but increases wealth accumulation as the precautionary motive for saving is strengthened, thus increasing participation. Gomes and Michaelides (2005) resolved the issue by assuming heterogeneity in risk aversion but at the cost that endogenously stock market participants are the more risk averse individuals which seems to contradict survey evidence that stock market participants tend to be more willing to take financial risk. [4] The present model by focusing on the risk properties of the stock investment itself rather than on risk aversion or background risk avoids the above mentioned contradiction.

Another appealing feature of this explanation emerges when the progressive social security system is considered. As it is shown in the paper, when replacement ratios are progressive and under the standard assumption of investment in a common stock index fund, conditional stock shares are larger for households with lower permanent income, in contrast to the empirical evidence. In

[4] The Survey of Consumer Finances asks a question about the willingness to take financial risk to earn higher returns. Econometric analysis by Bertaut and Starr-McCluer (2000), Curcuru et al. (2007) and Haliassos and Bertaut (1995) found that positive answers to this question correlate positively with stock market participation. To the extent that the answer to that survey question reflects risk aversion this suggests that it is the less risk averse individuals who participate in the stock market.

the present model wealthier households endogenously choose the stock portfolio with lower variance. This increases the share they optimally invest in the risky asset relative to poorer households, restoring the positive relationship between permanent income and conditional portfolio stock shares.

Given the key importance of heterogeneous stock portfolio diversification for the results of this paper it is important to insure that such an assumption is well motivated. This is the case both from an empirical and a theoretical perspective.

Empirical work on household portfolio diversification has traditionally relied either on survey data or on administrative records from brokerage houses or retirement plans. One example of the first type of studies is Polkovnichenko (2005). The author, using the SCF, finds that the median share of directly held stocks for equity holders declines with wealth except at the top of the distribution and that the number of directly held stocks increases from 1 in the bottom quintile to 15 in the top quintile of the distribution. Similarly, surveys about household stock market behavior conducted by the Investment Company Institute (1999 and 2002) show that the median number of individual stocks held is 2 for direct stock holders with less than 25000$ of financial wealth and 8 for those with more than 500000$; a similar pattern is observed when looking at stock mutual funds or at both types of equities jointly. The survey based studies, even though representative of the whole population and the whole household financial portfolio, have the limitation that they only allow to know the number of stocks or mutual funds held but not the variance of the risky portfolio, a more accurate measure of diversification since it captures also the correlation structure of stocks. In order to overcome this limitation other authors have used administrative records that also have the advantage of being subject to less reporting error. An example of this research is Goetzmann and Kumar (2008) who used the records of a large brokerage house and found large differences both in the number of stocks held and in estimated portfolio variances and even larger differences in risk-adjusted returns. As Bilias et al. (2008a) pointed out though, the behavior of investors with brokerage accounts is not representative of the whole population. Beside that, studies based on administrative records also do not cover the entire household financial portfolio. The limitation of both approaches have been overcome in recent research by Calvet et al. (2007 and 2008). The authors exploit a data-set collected by the Swedish statistical agency that covers the whole population and has information at the individual asset level, thus allowing to compute household portfolio performance. They find that idiosyncratic volatility is an important part of portfolio volatility and that there is a wide dispersion in both portfolio volatility and return losses caused by under-diversified investment. They also find that larger wealth and better education predict improved diversification of risky portfolios although the total dollar cost of under-diversification is higher for better diversified households due to more aggressive investment strategies.

From a theoretical perspective under-diversification arises in a variety of contexts. Brennan (1975) showed in a static model that in the presence of a fixed cost of transacting in each security the optimal number of risky securities in a portfolio is increasing in wealth hence the total variance of portfolio return is declining. More recently Van Nieuwerburgh and Veldkamp (2008) showed that in a model where agents allocate capacity to acquire information about securities the optimal portfolio implies investment in a diversified fund and at the same time in a small number of correlated securities. More capacity leads to larger investment in the under-diversified portfolio, however because of the information acquired the resulting portfolio displays better performance. Finally Polkovnichenko (2005) shows that investment in an under-diversified portfolio of stocks also arises under a class of preferences known as rank dependent utility although in this case there is no clear relationship between under-diversification and observable households' characteristics.

The papers mentioned above focus on the choice of assets in households' financial portfolio. They do that at the cost of simplifying the study of their real actions. [5] The present paper follows a complementary approach: it focuses on those real actions and how they interact with under-diversification of financial portfolios to derive predictions that can be quantitatively contrasted with the data. In doing so it simplifies the study of how the stock portfolio is constructed and why agents may choose not to diversify.

The rest of the paper is organized as follows. In Section 2 I present the description of the life-cycle model. In Section 3 I describe the choice of parameters, in Section 4 I describe the results of the quantitative analysis and in Section 5 I present some brief conclusions. Finally an Appendix describes the numerical methods used in the solution of the model.

[5] In particular they consider static problems where a consumption-saving decision is abstracted from and where no consideration is given to earnings uncertainty.

2 The Model

2.1 Demographic Structure and Preferences

Time is discrete and the model period is assumed to be one year. I let t be the number of periods an agent has spent in the model. Agents enter the model as workers at age 21 so that real-life age is equal to model age plus 20. Every agent can live up to a maximum of $T = 69$ periods, corresponding to age 89. I allow for uncertain life-span by assuming that in every period there is a positive probability $1 - p_{t+1}$ that the agent dies. All agents retire after G periods of life in the model provided they have not died before; the value of G is chosen so that agents retire at real age 65.

Agents value consumption but not leisure. Period utility is defined by a standard utility index $u(c_{i,t})$ and discounted at the rate β.

2.2 Labor and Retirement Income

Investor's i labor income after t periods of life in the model is given by:

$$log(y_{i,t}) = \theta_i + f(t) + z_{i,t} \tag{1}$$

for $a < G$. This formulation implies that there are three components that determine individual earnings. The first component denoted with θ_i is specific to the individual and fixed for the entire life time; it can be thought of as representing his earning ability as determined by education and other factors like genetics or the environment. The second component $f(t)$ is a deterministic function of age that is common to all individuals and is meant to capture the hump in life-cycle earnings that is observed in the data. Finally there is an idiosyncratic component $z_{i,t}$ which is assumed to follow an autoregressive process given by:

$$z_{i,t} = \rho z_{i,t-1} + \upsilon_{i,t} \tag{2}$$

where the innovations $\upsilon_{i,t}$ are $N(0, \sigma_{\upsilon}^2)$ and independent over time.

After retirement the agent receives a pension benefit $b(\theta_i, z_{i,G})$ that depends on his permanent earning type and the earnings shock in his last period of working life. This choice allows the model to capture some elements of the progressive US pension system without adding a further state variable.

The general notation for household income will be $Y_{i,t}$ where:

$$Y_{i,t} = \begin{cases} e^{ln(y_{i,t})} & \text{if } t \leq G \\ b(\theta_i, z_{i,G}) & \text{if } t > G \end{cases} \tag{3}$$

2.3 Financial Assets

In the economy there are three assets in which the agent can invest. First a one period risk-free bond with price q and return $R_f = 1/q$. Investment in this asset does not require payment of any cost. Second there are two mutually exclusive risky assets that I call "stock portfolio" with return R_t^j for $j \in \{l, h\}$ defined by the equation:

$$R_{t+1}^j - R_f = \mu + \varepsilon_{t+1}^j \tag{4}$$

where $\varepsilon^j \sim N(0, \sigma_{\varepsilon^j}^2)$ is an i.i.d. innovation and μ is the expected excess return common to the two stock investments. The variance of the two stock portfolios is related by the inequality $\sigma_{\varepsilon^h}^2 > \sigma_{\varepsilon^l}^2$.[6]

Investing in the stock market for the first time requires payment of an entry cost denoted F. This cost can be thought of as the cost needed to gather the

[6] It is assumed that the correlation between the returns on the two stock funds is zero at all leads and lags.

initial information about the stock market in general. Continued participation in the stock market requires payment of a per-period participation cost of F^h or F^l depending on whether the agent decides to invest in the stock portfolio h or l. The two per-period participation costs are related by the inequality $F^h < F^l$.

Summarizing, the agent can invest freely in the bond market. In order to gain access to the stock market he needs to pay an initial entry cost. Once this cost is paid, in every period the investor can decide not to pay any further cost and not participate in the stock market, to pay the small period fee F^h and invest in a poorly diversified stock portfolio with high variance $\sigma^2_{\varepsilon^h}$ or to pay the larger period cost F^l and invest in a well diversified stock portfolio with variance $\sigma^2_{\varepsilon^l}$. The small per-period cost F^h can be interpreted as the extra time cost of filling tax forms or the monetary cost of brokerage fees while the extra cost implied by F^l — the difference $F^l - F^h$ — includes various costs related to the construction of a well diversified stock portfolio like those needed to acquire the information on stock and mutual fund performance and the costs to trade in order to adjust the weights of those assets in the risky portfolio. These costs may also include psychological costs needed to overcome inertia and update past decisions. While part of these costs may be time costs or even purely psychological costs, I follow the tradition in this literature and treat all of them as monetary costs.

The amount of bonds and stocks that household i holds at time t is denoted with $B_{i,t}$ and $S_{i,t}$ respectively and it is assumed that

$$B_{i,t} \geq 0 \tag{5}$$

$$S^j_{i,t} \geq 0 \tag{6}$$

for $j \in \{l, h\}$. The two constraints have the usual meaning that the investor is prevented from borrowing against future labor income or retirement wealth and from selling short stocks.

2.3.1 Discussion

The menu of risky assets available to households is somewhat non-standard hence it requires some discussion. The two assets are meant to capture in a stylized way the idea of a well diversified and a poorly diversified portfolio of risky financial assets that the agent may construct by picking individual stocks and/or mutual funds shares. In particular the two assets should not be taken to be real assets like, for example, stock of company A and share of mutual fund B so that it is legitimate to treat them as mutually exclusive. This choice is motivated by the desire to generate endogenously and in a simple way the empirically observed fact that the performance of the risky financial portfolio held by different households is heterogeneous and that for many of them, especially less wealthy and less educated ones, it is well below the one on the stock market index. It should also be stressed that whenever there is a fixed cost to purchase, hold and trade in an asset, the optimal number of assets, hence the cost of holding the portfolio and its variance are inversely related which is what the assumption made here captures in a reduced form. Numerically, allowing the agent to choose simultaneously among n assets implies huge computational costs since these grow exponentially with the number of assets. The assumption made here that the two alternatives are mutually exclusive makes computational time grow linearly thus keeping it at a reasonable amount.[7] The next step in

[7] A more detailed discussion of the numerical problem is deferred to the Appendix.

the discussion is to explain the nature of the costs involved in holding an asset in the portfolio, especially in light of the existence of stock index funds that offer better diversification for lower management fees. Here we should observe that management fees are only a small part of the costs needed to trade in an asset. An agent willing to trade in an asset needs first of all to be aware of the existence of the asset and second to gather information about and understand its key properties like return, risk and fees. As it was pointed out in the recent literature on "rational inattention", even if information is in principle free or available at low cost, still it may require substantial efforts on the part of agents to process and absorb it. Further, agents may end up disregarding that information altogether because of constraints on their capacity to attend to it. [8] The costs of absorbing that information may be particularly relevant when it comes to forming financial portfolios both because the starting level of financial literacy for many investors seems to be rather poor and because the number of available options is very high. The first fact has been documented in several studies and is summarized in a paper by Lusardi and Mitchell (2007). Among else the two authors report that in a countrywide survey conducted in 2001 in the US only half of the respondents knew that mutual funds do not pay a guaranteed rate of return and just a little more than half knew that historically stock had outperformed bonds in terms of return. Also, using the 2004 Health and Retirement Survey the two authors found that among Baby Boomers aged 51-56, 80 percent did not even understand the principle of compound interest calculation. In a similar study conducted in Washington State, Moore (2003)

[8] A theoretical paper in the literature on "rational inattention" is for example Sims (2006). An application to consumer theory can be found in Reis (2006), one to business cycle theory in Mankiw and Reis (2007).

found that 57 percent of the respondents did not know the inverse relationship between interest rates and bond prices and about the same percentage did not know that a no load mutual fund involves trading costs. The second fact becomes apparent if one thinks about the large number of individual stocks and stock mutual funds available on the market. Overall the evidence reported above suggests that for a large part of the population financial decisions are complex decisions that involve a substantial effort to discover and assess the large number of alternatives available on the market. Some of these households even lack the tools to correctly assess those alternatives. Assuming that a larger monetary cost needs to be paid to access the better diversified fund is a convenient way to capture the costs of making well informed financial decisions, although the largest part of these costs may be time costs or even psychic costs. The idea of summarizing these costs related to information acquisition and processing by way of a monetary cost is quite common both in comparable life-cycle portfolio choice literature like Campell et al. (2001) and Gomes and Michaelides (2005) and in the new literature on "rational inattention"like Reis (2006). Also it should be observed that while here the cost is assumed to be monetary to follow the most common approach, the same result would be obtained if I assumed a direct penalty on utility to represent the time and psychic costs to attend to the information needed to form a well diversified portfolio of equities.

2.4 The Investor's Optimization Problem

In this subsection I will describe the optimization problem for a life-cycle investor. The Bellman equation for an age t, type θ agent is given by:

$$V^{t,\theta}(w_t, z_t, I_t) = \max_{c_t, B_{t+1}, S^l_{t+1}, S^h_{t+1}} \left\{ u(c_t) + \beta p_{t+1} V^{t+1,\theta}(w_{t+1}, z_{t+1}, I_{t+1}) \right\}. \quad (7)$$

In the equation above w_t is the amount of financial resources available to the agent at the beginning of the period. The variable z_t is the persistent component of labor earnings and $I_t \in \{0, 1\}$ is the index representing payment of the entry cost, with the usual convention that $I_t = 1$ means that the cost was paid before. The maximization is performed with respect to consumption and the amount of bonds and each of the stock portfolios the agent buys to carry to the next period. Given the description in the previous subsection the following restriction applies to the amount of the stock portfolios that can be bought:

$$\begin{cases} S^l_{t+1} > 0 & \Rightarrow S^h_{t+1} = 0 \\ S^h_{t+1} > 0 & \Rightarrow S^l_{t+1} = 0 \end{cases} \tag{8}$$

The maximization is performed under the constraints (5) and (6), the budget constraint (9) and the law of motion of financial resources (10) below. The budget constraint reads:

$$c_t + qB_{t+1} + S^l_{t+1} + S^h_{t+1} \leq w_t + Y_t - I^l_t F^l - I^h_t F^h - (1 - I_t) I^j_t F. \tag{9}$$

In the equation above the indexes I^l_t and I^h_t take a value of 1 if the agent pays the fixed per-period cost required by that particular stock portfolio and zero otherwise and $I^j_t = max\{I^l_t, I^h_t\}$. It follows from the description of the structure of financial assets in Section 2.3 that if $I^l_t = 1$ then $I^h_t = 0$ and vice-versa. With this in mind we see that the resource constraint states that the expenditure in consumption, bonds and stocks cannot exceed the sum of financial wealth and income from labor or social security net of payment of the costs of participating in the stock market if the agent decides to do so. In turn these costs are equal to $F + F^j$ if participation occurs for the first time in the agent's life and it is F^j if the agent had participated before. Labor and pension income Y_t are given by equations (1), (2) and (3). The law of motion of financial resources is given by the following equation:

$$w_{t+1} = B_{t+1} + R^l_{t+1} S^l_{t+1} + R^h_{t+1} S^h_{t+1} \tag{10}$$

that simply states that financial resources at time $t+1$ are equal to the sum of the realized return on bonds and stock portfolios. Finally the state variable in the investor's dynamic programming problem describing payment of the initial entry cost evolves according to the following rule:

$$I_{t+1} = I_t + (1 - I_t) max\{I^l_t, I^h_t\}. \tag{11}$$

3 Parameter Calibration

3.1 Preference Parameters

Preferences in the model are defined by two parameters. First the period utility index is of the standard iso-elastic form $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and the coefficient of relative risk aversion γ is taken to be 5, 7 or 9 with the mid value as the benchmark. These values fall in the range normally chosen in this literature. Second β is set at the value of 0.92 a number between the typical values found in the general equilibrium macroeconomic literature and those of partial equilibrium models of precautionary savings like Deaton (1991) and Carroll (1992).

3.2 Labor Income Process and Pensions

In order to fully characterize the labor income process we need to specify two different sets of parameters. First I fix the function $f(t)$ that describes the deterministic life-cycle profile of earnings. This is taken from the profile estimated by Cocco, Gomes and Maenhout (2005) for high-school graduates; when aggregated over five year periods the profile is also consistent with the one estimated by Hansen (1993) for the general population. Second we need to specify permanent earnings differences and the stochastic process that determines the yearly evolution of household earnings. To do that, first I fix the standard deviation of the innovation $\upsilon_{i,t}$ to the value of 0.025 which is consistent with the different estimates available for AR(1) processes of earnings. Then I fix the permanent component of individual earnings: I assume that θ_i can take two values and choose them so as to match the Gini index of earnings for first year workers. Finally I set ρ, the autocorrelation coefficient of the AR(1) process of earnings to 0.97 so that I can match the Gini index of earnings in the general population.[9] Interestingly and reassuringly the resulting autocorrelation coefficient is very close to the ones directly estimated on PSID data by Hubbard et al. (1994) and Storesletten et al. (2004).

In order to calibrate the social security payment I proceed in two steps. First I compute the average life-time earnings conditional on an agent's type and last year of work earnings. This forms the base used to compute the pension benefit the agent receives during retirement. Second the formula used in the US economy is applied to this average lifetime earnings. This formula fixes two bend points at 0.20 and 1.24 times average earnings and attributes a benefit that is 90 percent of earnings up to the first bend point, 32 percent from the first to the second and 15 above that. Retired households also receive social security payments in the form of medical and hospitalization benefits that are independent of their earnings history, so that I also add a fixed component to

[9]The values of the standard deviation of the innovation to labor earnings are taken from Hubbard, Skinner and Zeldes (1994) and Hugget and Ventura (2000); the Gini index of earnings to be matched comes from Díaz-Giménez et al. (1997).

the benefit and set it approximately equal to 19 percent of average earnings. [10]

3.3 Asset Returns and Participation Costs

I assume that the constant return to bonds R_f is 2% and that the average equity premium is 4% a value that is somewhat below the historical one but is the one commonly used in this literature (e.g. Campbell et al. (2001), Cocco et al. (2005) or Gomes and Michaelides (2005)). As far as the standard deviation of the risky return is concerned I fix its base value at 16% a value consistent with the historical evidence about the volatility of the U.S. stock market index. This value corresponds to the variable denoted with σ_l and will be used both as the standard deviation of the single available stock portfolio in the models with homogeneous diversification and as the standard deviation of the diversified stock portfolio in the model with heterogeneous diversification. In the latter model one more parameter must be defined, that is, the standard deviation of the poorly diversified portfolio denoted with σ_h in the section describing the model. In order to calibrate this parameter I will use the results in Calvet at al., (2007, 2008). The three authors report the distribution of the standard deviation of risky portfolio — that includes by definition individual stocks as well as stock mutual funds — of a representative sample of Swedish households. They find a median standard deviation of 20.7 %, a ratio between the 75^{th} to 25^{th} percentile of standard deviations of 1.43 and a ratio between the 90^{th} and the 10^{th} percentile of 2.53.[11] Based on these data I will present a baseline calibration where σ_h is set at 32 % or double the one on the diversified portfolio and then consider in the sensitivity analysis two more cases: one where $\sigma_h = 1.5 \times \sigma_l$ or

[10]The figures used are taken from Huggett and Ventura (2000).

[11]These numbers are based on Table 4 in Calvet et al. (2007).

24 percent and one where $\sigma_h = 2.25 \times \sigma_l$ equivalent to 36 percent.

Next we have to calibrate the three different costs that agents face to participate in the stock market. The values are chosen with reference to wealth in the model. More specifically I set F to 3.5 percent of median wealth, F_h to 2.3 percent of median wealth and F_h to 0.23 percent of that. Using the figure reported in Kennickell at al. (2000) that median financial wealth in the U.S. was 22.6 thousands dollars in 1998, we can see that in monetary terms the fixed entry cost is equivalent to 785 dollars, the high per period participation cost needed to achieve a well diversified portfolio is about 500 dollars while the low per period cost needed to buy the under diversified stock portfolio is equivalent to about 50 dollars.[12] There is no empirical estimate of the initial entry cost F but the monetary equivalent of the value used here is a little below the one used for example by Gomes and Michaelides (2005). There have been instead efforts to estimate the per period participation cost. Work by Vissing-Jørgensen (2002 and 2003) and by Paiella (2001) have found values between 50 and 200$. These costs are often interpreted as the value of the extra time cost of filling tax returns when the household holds stocks and/or the amount of brokerage fees. The value used here for the low per period cost lies at the lower bound of that interval. The value used for the high per period cost is 2.5 times the upper bound of that interval. This choice seems reasonably low given the fact that in the interpretation given here several elements, not included in the estimates

[12]The comparison to financial wealth to give a dollar equivalent to the costs used in the model is the most appropriate given that in the model all wealth is financial. Use of median wealth reflects the fact that the model cannot reproduce well the wealth distribution at the top and it is well known that the top wealthy agents affect disproportionately mean values of wealth but not the median.

quoted above, contribute to it: the monetary costs to actually perform transactions, the time costs to gather information about the asset return processes and their weight in the portfolio plus those needed to evaluate alternative investment options and carrying out the decision. Finally included are also the psychological costs needed to overrun inertia or status quo biases in decision making. Indirect evidence that these latter, hard to quantify costs, may be large can be found in empirical studies on the frequency of trade: for example Ameriks and Zeldes (2004) found in the TIAA-CREF sample that reallocations of contributions in the fund are very rare despite the actual monetary and time cost of performing the transaction is virtually zero; moreover when those changes do occur they tend to be large. Also, while studies of administrative records of brokerage houses often found considerable trade, Bilias at al. (2008a) show that those data are not representative of the whole population and that if one looks at surveys like the SCF that cover the whole population infrequency of trade emerges as widespread. Once again this finding suggests that the cost to adjust the stock portfolio may be quite larger than the pure monetary cost of the needed transactions.

4 Results

Results are organized into three subsections. In the first one I start with a benchmark parametrization and consider the standard case where there is only one possible risky investment with the variance of the return matching the one on the stock market index. Then, using the same set of parameters, I introduce the possibility of investing in the two mutually exclusive risky assets with different variance. Results in these two subsections cover decision rules, the life-cycle profiles of the variables of interest and the corresponding population averages. In the second subsection I report results of a sensitivity analysis on the risk aversion coefficient and the difference between the variance of the return on the two stock funds. Finally in the last subsection I present a discussion of two issues: one is the interaction between retirement and the financial decisions of households in the model and the second is a qualitative description of the effects of introducing housing in the model.

4.1 Baseline Results

4.1.1 Decision Rules and Life-Cycle Profiles: The Model with Diversified Stock Investment

In this section I report the results for the benchmark model where only one risky asset is available to the investor. The risk aversion coefficient is set at 7 in this case. In Figure 1 I report a sample decision rule. The function represents the portfolio weight of stocks for a low earning type agent in his fifth year of working life that had previously paid the initial entry cost. The three lines refer from bottom to top to the lowest, median and highest labor earnings shock. These decision rules are quite standard: for very low levels of wealth the agent does not pay the per period participation cost hence does not invest in the stock market. Once wealth increases beyond a certain threshold he will pay the cost and invest all of his wealth in the stock market, then as wealth further increases the optimal share of stocks declines. Also for any level of wealth the optimal share of stocks increases in the current labor earnings shock, reflecting the fact that with a highly persistent labor earnings process the agent has a larger implicit holding of relatively safe human wealth.

Figures 2 and 3 report the life-cycle profiles of participation rates and conditional stock shares by ten year age groups. Profiles are reported for the average population by the thick continuous line and separately by permanent earning types by the thinner dash and dash-dot line. As it can be seen participation rates are low at young ages, since agents have not yet accumulated enough wealth to make it convenient to pay the initial entry cost. As households move into mid age and saving for retirement picks up, the participation rate increases up to a value of about 90 percent. After retirement, as agents start to draw down their wealth to finance retirement consumption the participation rate slowly declines to about 60 percent. As expected the participation rate for high ability types is higher than the average and that of low ability types is lower with a difference that becomes non negligible late in life. The appreciable difference in participation rates between types and the decline after retirement is a consequence of the per period participation cost: as agents move into their old age their asset holdings are reduced to an extent that even a very small cost will deter them from participating and this effect is stronger for low earning ability types that hold less wealth to start with.

If we look at Figure 3 we see that conditional portfolio shares of stocks start at 100 percent in the first decade of life and then monotonically decline to about 70 percent at retirement to remain roughly constant until the end of the life cycle. The other notable feature is that the average conditional share is larger for low earning types than for high earning types. When analyzed over the life cycle this result comes from shares that are roughly equal early in life but that are quite higher for the low types later and especially after retirement. The interpretation of this outcome of the model follows from the result presented in

Cocco et al. (2005) that non financial income acts as a better substitute for the risk free bond. With progressive social security high permanent income agents face lower replacement ratios so that they have to rely more on financial income to finance their post retirement consumption, hence they will optimally choose to reduce their exposition to stockholding risk. This implication of the model is at odds with several empirical studies that find that proxies for permanent income like education have a positive effect on conditional stock shares. [13]

4.1.2 Decision Rules and Life-Cycle Profiles: The Model with Diversified and Under-Diversified Stock Investment

In this section I consider the effects of heterogeneous under-diversification under the assumption that risk aversion is 7 like in the previous section. As anticipated in the description of the model now agents have the possibility to invest in either of the following two stock portfolios: one entails a small per period participation cost but a high return variance and the second entails a higher per period participation cost but a lower return variance. The standard deviation of the poorly diversified stock portfolio is twice that of the diversified one.

Figure 4 shows the decision rules for this case. For comparison with the previous case I consider again an agent that has been 5 periods in the model, who is a low permanent income type and who has already paid the initial entry cost. The decision rules now show a more complicated pattern. For low levels of wealth the agent does not want to pay even the small cost associated to the undiversified stock portfolio hence does not participate. Compared to the baseline model the threshold level of assets that triggers participation is slightly

[13] This result can be found for example in Haliassos and Bertaut (1995), Kennickell et al. (2000) and Wachter and Yogo (2008).

larger, moreover the optimal stock share past that threshold is lower and declines much faster. All these effects reflect the fact that in this version of the model if the agent pays the small per period cost F^h then he can invest only in a poorly diversified stock portfolio. The increased variance reduces the optimal share and indirectly also increases the amount of wealth that is needed to induce participation.[14] At substantially greater levels of assets we can see that there is a discontinuity in the optimal stock share which jumps up by 30 or more percentage points. The explanation of this discontinuity is the following. At each level of wealth the optimal weight of stock is larger if stock investment occurs in the well diversified fund than if it occurs in the poorly diversified one. Once the level of wealth is sufficiently large the benefits of this larger total investment in the stock fund outweigh its cost hence it becomes convenient to pay it. After the threshold is passed the agent then invests in an asset with a lower volatility so the optimal share increases. For further increases in the wealth level the decision rule is again declining and follows closely the pattern of the decision rule of the baseline case.

The effects of this change in the decision rules when the model is simulated are represented in Figures 5 and 6. We can see from Figure 5 that the participation rate still shows a hump shape over the life cycle. When compared to the baseline case though the average profile shifts down between 20 and 30 percentage points depending on the age group. Most of this shift in the average profile is determined by an even larger movement of the low permanent income types, since the other group is barely affected. The reason of this change is

[14]The threshold level that induces participation may not seem that larger than in the baseline case, however it would be at older ages. Plots of the decision rules at older ages are available from the author upon request.

that now unless agents are willing to pay the larger per period cost they will have to invest in a poorly diversified stock portfolio. The increased variance and reduced optimal share reduce the benefits of doing so, keeping a larger fraction of agents away from the stock market. This is especially true for the low ability types who accumulate less wealth since they face both lower earnings and more generous social security replacement ratios.

Comparing Figure 6 with Figure 3 we see that the introduction of under-diversification has a large effect on stock shares conditional on participation as well. The thick continuous line that represents the population average profile moves down at all ages and by an amount that is about 20 percentage points from the first decade of life until around retirement and then up to about 40 percentage points in the last decade of life. Beside that we can see that in the model with heterogenous diversification the conditional stock share is larger for the high permanent income types overturning the result obtained in the case of fully diversified stock portfolios. This can be seen from the fact that the life-cycle profile of high types is above that of low types both at young and old ages and is virtually the same in the decade where retirement falls. The intuition is that households of the high permanent income type are more likely to hold wealth levels in the range where it becomes convenient to pay the larger per period cost and invest in the well diversified stock fund. For this reason while their larger ratio of financial wealth to human wealth would induce them to reduce their exposure to stock market risk, the fact that they have more resources, hence find it convenient to invest in the lower variance stock portfolio creates an opposite force that in fact makes them invest a larger proportion of their wealth in the stock market.

4.1.3 Participation Rates and Conditional Shares: Models and Data

In this section I compare the model results with the data. This is done first for the general population average of both participation rates and conditional shares and then for the corresponding life-cycle profiles. Population averages are shown in Table 1. The first row of the table reports that in 1998, 48.9 percent of U.S. households invested directly or indirectly in equities and for those households that did participate in the stock market on average 59.6 percent of financial assets was held in stock. [15] The second row of the table reports that in the basic model where all agents invest in a diversified stock portfolio with a 16 percent standard deviation of returns, the participation rate is 76.5 percent and the allocation to stocks for participants is 79.7 percent. In the final row we see that when it is costly to achieve a diversified stock portfolio the participation rate drops by 25 percentage points and the conditional share drops by about 15 percentage points. As a consequence the participation rate in this model is 51.3 percent and the conditional share is 64.2 percent, that is, about 3 and 5 percentage points above the empirical counterparts.

Not reported in the table, but worth mentioning, are the figures disaggregated by types: for the low type the participation rate drops from 67.6 percent to 25.9 percent while for the high types the drop is only from 85.5 percent to 77.9 percent. Conditional stock shares drop from 82.1 to 56.6 percent for the low types and from 77.9 percent to 66.7 percent for the high types. As we could infer from the life-cycle profiles moving to a model where poor diversification of the stock investment is allowed has the effect that high types will allocate

[15] The figures reported are taken from Guiso et al. (2001) and are based on the Survey of Consumer Finances.

to stocks about 10 percent more of their wealth while when a well diversified portfolio can be constructed at no extra cost they would allocate 4 percent less compared to low types. The difference in stock allocation by types in the heterogeneous diversification model is consistent with the figures reported by Wachter and Yogo (2008) for the differences in conditional stock shares by education — a proxy for permanent income — in the SCF.

The comparisons between life-cycle profiles in the model and in the data are performed in Table 2 and 3. Those tables report the data underlying Figures 2, 3, 5 and 6 respectively, except that, in order to make the comparison with the data, the oldest two groups of the population are lumped together in the 70+ year old group. The source for the empirical figures is the study by Guiso et al. (2001) mentioned above. As it can be seen in the first row of Table 2 the participation rate is hump shaped in age in the data: it starts at 34.3 percent in the youngest age group, climbs up to 61.4 percent in the 50 to 60 year group and then declines to 32.4 percent in the oldest segment of the population. This inverted U-shape can be found in both models as well. However while the model with homogenous diversification generates a profile that is systematically quite above the one in the data, the model with under-diversification is much closer to the data: as we can see by comparing the first with the third row of Table 2 the model with heterogeneous diversification matches the data quite well between age 30 and 60, after that the decline is somewhat more pronounced in the data than in the model. Looking at the first row of Table 3 we can see that the cross section view of the data shows a life cycle profile of conditional stock shares that is almost constant. In both models the life-cycle profile of conditional stock shares is declining as previous models in the literature have found. There are important differences between the model with homogeneous and heterogeneous diversification though. In fact the model with costly diversification leads to a substantial reduction in conditional shares early in life: in the 20 to 30 years of age group they are about 81 percent compared to virtually 100 percent in the model with fully diversified stock portfolio only. As a result, in the model with costly diversification the profile of conditional stock shares declines only by about 15 percentage points between the first decade of life and the 50 to 60 year group, thus moving a step towards a closer match with the data when compared with the baseline model that displays a 26 percentage point decline. In the two oldest groups of the population though, conditional shares in the model with heterogeneous stock diversification show a further decline while in the model with homogenous stock investment they remain virtually constant.

Summarizing the results of the comparison with the data performed in this section we can reach the following conclusions. Allowing for individual portfolios of risky assets to be less than fully diversified allows the model to generate moderate participation rates and conditional stock shares that are close to the data. Both models do a good job at reproducing qualitatively the hump shaped life-cycle profile of participation rates that is observed in cross sectional data, although quantitatively the model with heterogeneous diversification fares significantly better. Both models generate a declining pattern of conditional stock shares over the life-cycle that is not in line with the data. When we break the life cycle into two broad periods we see that adding under-diversification moves the model closer to the data in the first part of life — approximately up to retirement — but moves it away from them for ages above 70.

4.2 Sensitivity Analysis

In this section I perform a sensitivity analysis on some parameters of the model, more precisely the coefficient of relative risk aversion and the difference between the standard deviation of the under diversified portfolio and the diversified one. Given that the shape of life-cycle profiles does not change significantly across the different experiments these are not reported and the focus will be on population averages either aggregate or by type.

In the first set of sensitivity analysis I fix the difference in the volatility of the return on the two portfolios to the one used in the baseline case and then consider two more values of the coefficient of relative risk aversion, namely 5 and 9. Results are reported in Tables 4 and 5. In the first row of each panel of Table 4 I report the participation rate in the model with homogeneous variance of returns while in the second row the one of the model with heterogeneous variance. As it can be seen, increasing risk aversion from 5 to 9 increases the participation rate from 69.4 percent to 80.5 percent in the model with one stock portfolio and reduces it from 53 percent to 50.1 percent in the model with two stock portfolios. To understand this result observe that increasing the risk aversion coefficient has two effects on the incentives to participate that are opposite in sign: it reduces the optimal share of wealth invested in stocks but it increase wealth accumulation for precautionary reasons. When agents face a larger volatility of returns optimal shares are more sensitive to changes in the coefficient of relative risk aversion hence the first of the two effects is stronger and as a result participation varies negatively with the coefficient of relative risk aversion. This intuition is confirmed if we look at results disaggregated by permanent income types: as we can see, when γ moves from 5 to 9 the participation rate increases from 71.9 percent to 77.3 percent in the high income group but declines from 34 percent to 23.9 percent in the low income group. This is because low income agents are those who mostly face the poorly diversified stock portfolio.

Looking at Table 5 we can see that the coefficient of relative risk aversion has a qualitatively similar effect in both models: the conditional share declines from 93.5 percent to 64.9 percent in the homogeneous diversification model and from 81.1 percent to 46.3 percent in the heterogeneous diversification one when moving γ from 5 to 9. As the figures just mentioned suggest the effect of introducing under-diversification is then quantitatively dependent on the risk aversion coefficient: this factor brings down the conditional share by almost 20 percentage points if γ is equal to 9 and by 12 percentage points if γ is equal to 5. Looking at the second and third column of the table we see that for all values of the risk aversion coefficient considered here, in the heterogeneous diversification model the high permanent income types on average hold a share of stock between 10 and 15 percentage points larger than that of the low income types while in the homogenous diversification model it is always true that it is the latter group who holds a marginally larger share of their wealth in stocks.

In the second set of sensitivity analysis I fix the value of the risk aversion coefficient to 7, that is, the one used in the benchmark analysis. I then consider 3 different values of the standard deviation of the poorly diversified stock portfolio, namely 1.5, 2 and 2.25 times that of the diversified one. Results are reported in Tables 6 and 7. If we look at Table 6 we see that the participation rate increases from 47.7 percent, when we assume that the standard deviation of the poorly diversified stock portfolio is 2.25 times that of the diversified one, to 65.9 percent when the ratio of the two standard deviations is 1.5 only. The explanation is that a reduction in the variance of the under-diversified portfolio increases the optimal share invested in it at all levels of wealth. This reduces the threshold level of wealth that triggers participation hence increases the participation rate. If we look at the second and third column of the table we see that the increase in participation rates is especially strong among low permanent income types: it is 22.7 percent when σ_h is 2.25 times σ_l and it climbs up to 48.4 percent when the ratio between the two is 1.5. This is a consequence of the fact that low types accumulate less wealth hence more often invest in the low cost, poorly diversified stock portfolio. When we move to Table 7 we see an interesting result: the conditional share is quite insensitive to the standard deviation of the under-diversified stock portfolio. When this is moved from 1.5 to 2.25 times that of the diversified portfolio the conditional share moves only from 63.3 percent to 65.1 percent. The fact that the conditional share is robust to changes in the volatility of the poorly diversified stock fund stems from two opposing forces. On the one hand a higher variance of the under-diversified portfolio reduces the optimal share of stocks invested in it. On the other hand it increases the benefits from holding the diversified stock portfolio, lowering the threshold level of wealth that determines the choice of paying the larger per period cost associated with it. As a result while those agents who still do not find it optimal to invest in the well diversified portfolio will hold a smaller share of stocks, a larger proportion of agents will invest in the diversified one and for this reason invest a larger share of wealth in stocks. The two effects approximately cancel out as the table shows. This interpretation is confirmed when we look at the second and third column of the table. Here we can see that while the conditional share marginally declines as the variance of the under-diversified portfolio increases for the high types, for the low types it increases from 52.6 percent to 59.8 percent reflecting a greater importance of the composition effect.

We can summarize this section by saying that in all cases considered recognizing the potential role of under-diversification of individual stock portfolios has sizeable effects on both participation rates and conditional stock shares, moving the model closer to the empirically available estimates. When compared with an equally parametrized standard model with homogenous diversification the reduction in participation rates ranges between 14 and 32 percentage points and the reduction in conditional stock shares ranges from 12 to 18 percentage points. Beside that we see that consistently with the data the model with heterogeneous diversification in stock portfolios generates conditional shares of stocks that are between 7 and 17 percentage points higher for high permanent income types than for low permanent income types while the basic model would generate the qualitatively opposite pattern, against what the empirical evidence suggests.

4.3 Discussion

In this section I will discuss two issues: one are the interactions between retirement and financial decisions in the model and the second are the likely effects of assets that albeit relevant in actual household portfolios are omitted in the present research.

4.3.1 Retirement and Household Financial Decisions

The interaction between retirement and household financial decisions is a two way relationship. On the one hand we have the effects that social security provisions have on households' financial decisions. As we saw in the previous sections if all agents invest in the same perfectly diversified stock fund and under the assumption that social security replacement ratios are progressive, low permanent income agents should invest a larger proportion of their wealth in the stock market than high permanent income agents, something that is against the empirical evidence. This is because, as it was pointed out by Cocco et al. (2005) human wealth is a better substitute for a risk-free asset. If social security benefits are progressive high earning households will have larger financial to human wealth ratios hence they will try to reduce their exposition to stock market risk more than low earning households. The results in the present research showed that this result is reversed when it is costly to construct a well diversified stock portfolio: in this case agents that accumulate more financial wealth will have the resources and the incentives to diversify better hence they will choose to invest larger shares in the stock market.

In the opposite direction, household financial choices will determine their wealth, hence their preparedness for retirement. If it is costly to construct a well diversified stock portfolio, poorer agents may be more easily driven out of the stock market or they may reduce their exposition to it, giving up the benefits of the equity premium. This may have adverse distributional consequences and affect negatively preparedness for retirement of those who are already in the left tail of the wealth distribution. This issue gains special importance in times when social security systems appear to be unsustainable and more is asked on the part of households to provide for their old age. This potential implication of costly diversification can be tested with the model presented in this research. In Table 8 I present the share of wealth of a few percentiles of the wealth distribution and the associated wealth levels. The figures reported in the table refer to agents in the year prior to retirement.[16] What we can see from the table is that the prediction outlined above is confirmed by the model results, at least qualitatively. When the construction of a well diversified portfolio is costly the share of wealth of the bottom 40 percent and 60 percent of the population declines from 4.5 to 3.1 percent and from 15 to 12.6 percent respectively. On the contrary the share of the top 20 and 10 percent wealthier agents increases from 63.1 to 65.3 and from 43 to 44.6 percent respectively. As we can see these numbers are quantitatively modest, except perhaps for the bottom group. This result is confirmed when we look at the change in the wealth level at the percentiles of the wealth distribution that mark the boundaries between the wealth cells defined above. It is possible to see that except for the 40^{th} percentile where wealth is reduced by about a third — from 7.9 to 5.4 — when under diversification is introduced in the model, the changes in asset holdings are negligible. In order to understand why the effect of under-diversification on the wealth distribution is not large one should observe that the value of the per period cost needed to buy the well diversified stock fund was indeed small, hence it could affect the demand of risky assets only at relatively low values of wealth. Around retirement asset holdings are at a maximum so that except for agents that are in the left tail of the distribution the cost needed to purchase the well diversified portfolio is affordable, consequently under diversification will not be

[16]The issue of wealth inequality at retirement has been studied in a quantitative model for example in Hendricks (2007). In that model the focus is on heterogeneity in discount factors while here it is on heterogeneity in investment opportunities. The role of differential access to the stock market on wealth inequality has been studied empirically in Bilias, Georgarakos and Haliassos (2008b).

a problem for a majority of households. [17]

4.3.2 Other assets

Following the vast majority of the literature about life-cycle portfolio choice in incomplete market economies I have focused this research on the choice between a risk free and a risky financial asset omitting other assets like business assets and housing that are an important part of portfolios. With respect to business assets their omission is not very relevant in the present context since business owners represent about 10 percent of the population so that the results of the model presented here would still be valid for the vast majority of the population hence a fortiori for global averages. On the contrary housing is more widespread hence is likely to have a larger impact. First housing wealth substitutes financial wealth directly and for this reason would reduce stock market participation rates for any level of the entry and participation costs. Second, according to the findings in Cocco (2005), housing price risk may crowd out stock holding risk, reducing optimal stock shares and through this indirect channel further reducing participation rates. Both effects, would work in the direction of further strengthening the conclusions of the present research, in the sense that lower diversification costs or differences in return variances would generate the same participation rates and conditional shares.

[17] One might wonder what would be the consequences of reducing replacement ratios, perhaps in the context of phasing out the pay-as-you go system. Although in principle this could lead lower wealth agents to accumulate more, hence buy more diversified stocks, dampening the distributional effects of the reform, we refrain from simulating that since such a reform would have wider general equilibrium effect that the partial equilibrium model considered here cannot capture.

5 Conclusions

In the present paper I have considered an extension of the basic life-cycle asset allocation model that allows for heterogeneous under-diversification. This was obtained in a stylized way by assuming the existence of two mutually exclusive stock portfolios with different return variances and different associated fixed participation cost. The result that emerged is that under-diversification of households' stock portfolios provides a quantitatively plausible explanation for two key empirical facts: the moderate stock market participation rates and portfolio share of stocks for participants. Moreover, in line with the empirical evidence, endogenously agents with greater permanent income and/or wealth invested in the more diversified stock. This allowed the model to reverse the pattern of portfolio stock shares by permanent income induced by the progressive social security formula bringing the model in line with the data along this dimension as well.

The hope is to have convinced the reader that under-diversification of stock investment is a key feature to understand households' life-cycle asset allocation. Heterogeneous under-diversification was obtained here in a stylized way; the natural next step is to integrate the basic life-cycle portfolio choice model with a more sophisticated theory of household portfolio diversification. Given the huge computational effort needed to solve such a model and the fact that there is not yet a consensus on which theory best explains under-diversification of stock portfolios, this research line is left for the future.

Appendix

A Numerical Solution Method

In this appendix I will give a brief and informal description of the numerical methods used to solve the model presented in the paper. In a life-cycle model we know that the value function at age $T+1$ is uniformly equal to zero. Then we can substitute the zero function in the right hand side of equation (7) and perform the maximization to get the decision rules and find V^T, the value function at age T. In the same way we can work backward up to age $t = 1$. Notice that because of the per-period participation cost, at each age we need to find three functions, V^{t,I_p}, representing expected continuation utility. Here the index I_p summarizes the three possible choices with respect to the stock market participation decision, that is: not participating, participating and buying the poorly diversified stock fund and participating buying the well diversified stock fund. Once the three functions have been computed the value function will be their upper envelope. [18] At each state space point the optimization must then be performed three times, first with respect to bonds only and then with respect to the risk free and risky asset jointly for each of the two possible risky assets and subtracting the appropriate participation costs in the budget constraint. The one dimensional optimization is performed by using Brent's algorithm; the two dimensional problem is solved by exploiting the fact that for any function $f(x, y)$ we can write $\max_{x,y} f(x, y) = max_x\{max_y f(x, y)\}$ and then applying Brent method along both dimensions. The advantage of Brent method is that it exhibits super-linear convergence, so it is faster than bisection — thus also

[18] To simplify notation here I omit the arguments of the V functions.

of direct search — but does not require concavity of the objective, a property that is violated here because of the fixed cost.[19] On the other hand it is slower than Newton like methods that cannot be applied here because of the lack of concavity mentioned above. Given the algorithm used here, computational costs grow exponentially with the number of optimization variables. Making the two risky assets mutually exclusive on the other hand implies that the computational cost of solving the investor's dynamic programming problem grows linearly with their number. Moreover making the two assets exclusive makes it possible to exogenously pick the variance of the two portfolios based on the data, avoiding a time consuming calibration to pick the parameters of the asset return processes and costs to generate those values endogenously. Those computational advantages justify the choice made here.

Once the decision rules are obtained they are simulated for 200000 agents over 6500 periods. Life-cycle profiles are then obtained by averaging over 100 cross-sections to smooth out the effects of a particular sampling history of stock return realizations.

[19] It does require single-peakedness of the objective function, which holds in the present model. See Brent (1973) for details.

References

[1] Ameriks, John and Zeldes, Stephen. 2004. "How Do Household Portfolio Shares Vary with Age." Working Paper, Columbia Business School.

[2] Bertaut Carol C. and Starr-McCluer Martha. 2000. "Household Portfolios in the United States". *Federal Reserve Board of Governors*, *Finance and Economics Discussion Paper*: 26 (April).

[3] Bilias Yannis, Georgarakos Dimitris, Haliassos Michael. 2008a. "Portfolio Inertia and Stock Market Fluctuations." Working Paper.

[4] Bilias Yannis, Georgarakos Dimitris, Haliassos Michael. 2008b. "Equity Culture and the Distribution of Wealth." Working Paper.

[5] Brennan Michael J. 1975. "The Optimal Number of Securities in a Risky Asset Portfolio When There are Fixed Costs of Transacting: Theory and Some Empirical Results". *Journal of Financial and Quantitative Analysis*, 61(4): 1553-1604.

[6] Brent R. 1973. "Algorithms for Minimization without Derivatives". Prentice-Hall, Englewood Cliffs, NJ.

[7] Calvet Laurent E., Campbell John Y. and Sodini Paolo. 2007. "Down or Out: Assessing the Welfare Costs of Household Investment Mistakes". *Journal of Political Economy.* 115 (5): 707-747.

[8] Calvet Laurent E., Campbell John Y. and Sodini Paolo. 2008. "Fight or Flight? Portfolio Rebalancing by Individual Investors". *Quarterly Journal of Economics.* Forthcoming.

[9] Campbell John Y. 2006."Household Finance." *Journal of Finance*, 61(4): 1553-1604.

[10] Campbell John Y., Cocco Joao, Gomes Francisco and Maenhout Pascal. 2001. "Investing Retirement Wealth ? A Life-cycle Model". In Feldstein Martin and Campbell John Y. (Eds.) "Risk Aspects of Investment-Based Social Security Reform". University of Chicago Press. Chicago, IL.

[11] Carroll Christopher. 1992. "The Buffer-Stock Theory of Saving: Some Macroeconomic Evidence." *Brookings Papers on Economic Activity*, 2: 61-156.

[12] Cocco Joao; 2005. "Portfolio Choice in the Presence of Housing". *Review of Financial Studies*, 18(2): 535-567.

[13] Cocco Joao; Gomes Francisco and Maenhout Pascal. 2005. "Portfolio Choice over the Life-Cycle". *Review of Financial Studies*, 18(2): 491-533.

[14] Curcuru Stephanie, Heaton John, Lucas Deborah and Moore Damien. 2007. "Heterogeneity and Portfolio Choice: Theory and Evidence". In Ait-Sahalia Yacine and Hansen Lars P. (Eds.) "Handbook of Financial Econometrics". Elsevier Science. Amsterdam.

[15] Deaton Angus. 1991. "Savings and Liquidity Constraints." *Econometrica*, 59: 1221-1248.

[16] Díaz-Giménez Javier, Quadrini Vincenzo and Ríos-Rull Jos-Victor. 1997. "Dimensions of Inequality: Facts on the U.S. Distributions of Earnings, Income and Wealth". *Federal Reserve Bank of Minneapolis Quarterly Review*, 21, (Spring): 3-21.

[17] Goetzmann William N. and Kumar Alok. 2008. “Equity Portfolio Diversification ”. *Review of Finance*. 12: 433-463.

[18] Gomes, Francisco and Michaelides Alexander. 2005. “Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence”. *The Journal of Finance*. 60(2): 869-904.

[19] Guiso L., Haliassos M. and Jappelli T. (Eds.) 2001. “Household Portfolios”. MIT press. Cambridge, MA and London:

[20] Haliassos, Michael and Bertaut Carol C. 1995. “Why do so Few Hold Stocks”. *The Economic Journal*. 105(432): 1110-1129.

[21] Haliassos, Michael and Michaelides Alexander. 2003. “Portfolio Choice and Liquidity Constraints”. *International Economic Review*. 44(1): 143-177.

[22] Hansen Gary D. 1993. “The Cyclical and Secular Behaviour of the Labour Input: Comparing Efficiency Units and Hours Worked”. *Journal of Applied Econometrics*. 8(1): 71-80.

[23] Heaton John and Lucas Deborah. 1997. “Market Frictions, Savings Behavior, and Portfolio Choice”, *Macroeconomic Dynamics*. 1: 76-101.

[24] Heaton John and Lucas Deborah. 2000. “Portfolio Choice in the Presence of Background Risk”, *Economic Journal*. 110: 1-26.

[25] Hendricks Lutz. 2007. “How Important is Discount Rate Heterogeneity for Wealth Inequality ?”. *Journal of Economic Dynamics and Control*. 31: 3042-3068.

[26] Hubbard Glenn; Skinner Jonathan and Zeldes Stephen. 1994. "The Importance of Precautionary Motives in Explaining Individual and Aggregate Saving". *Carnegie-Rochester Conference Series in Public Policy.* 40: 59-126.

[27] Huggett Mark. and Ventura Gustavo. 2000. "Understanding Why High Income Households Save More than Low Income Households". *Journal of Monetary Economics*, 45: 361-397.

[28] Investment Company Institute. 1999 and 2002. "Equity Ownership in America". (New York, NY and Washington, DC)

[29] Kennickell Arthur B., Starr-McCluer Martha and Surette Brian J. 2000. "Recent Changes in U.S. Family Finances: Results from the 1998 Survey of Consumer Finances". *Federal Reserve Bulletin*: 86 (January).

[30] Lusardi Annamaria and Mitchell Olivia S. 2007. "Financial Literacy and Retirement Preparedness: Evidence and Impications for Financial Education". *Business Economics*. January: 35-44.

[31] Mankiw Gregory N., and Reis Ricardo. 2007. "Sticky Information in General Equilibrium". *Journal of the European Economic Association.* 5 (2-3): 603-613.

[32] Moore Danna. 2003. "Survey of Financial Literacy in Washington State: Knowledge, Behavior, Attitudes and Experiences ". Tecnhical Report n. 03-09, Social and Economic Sciences Research Center, Washington State University.

[33] Paiella P. 2001. "Limited Financial Market Participation: A Transaction-Cost Based Explanation". The Institute for Fiscal Studies. WP 01/06.

[34] Polkovnichenko Valery. 2005. "Household Portfolio Diversification: A case for Rank Dependent Preferences". *Review of Financial Studies*, 18(4): 1467-1501.

[35] Reis Ricardo. 2006. "Inattentive Consumers". *Journal of Monetary Economics*. 53 (8): 1761-1800.

[36] Sims Christopher. 2006. "Rational Inattention: Beyond the Linear-Quadratic Case". *American Economic Review*. 96(2): 158-163.

[37] Storesletten Kjetil; Telmer Chris I., and Yaron Amir. 2004. "Cyclical Dynamics in Idiosyncratic Labor-Market Risk". *Journal of Political Economy*. 112(3): 695-717.

[38] Van Nieuwerburgh Stijn and Veldkamp Laura. 2008. "Information Acquisition and Portfolio Under-Diversification ". Working Paper.

[39] Vissing-Jørgensen A. 2002. "Towards an Explanation of Household Portfolio Choice Heterogeneity: Nonfinancial Income and Participation Cost Structures". National Bureau of Economic Research, Cambridge, MA, Working Paper 8884.

[40] Vissing-Jørgensen A. 2003. "Perspectives on Behavioral Finance: Does "Irrationality" Disappears with Wealth? Evidence from Expectations and Actions ", in Mark Gertler and Kenneth Rogoff eds. NBER macroeconomics Annual, MIT Press, Cambridge, MA.

[41] Wachter Jessica A. and Yogo Motohiro. 2008. “Why Do Household Portfolio Shares Rise in Wealth ? ” Working Paper.

Table 1

Average participation rates and conditional shares.

Table 1 reports the average participation rates and conditional stock shares. The top line refers to the US data from the 1998 SCF and is based on Guiso et al. (2001). The second and third line report respectively the values from the baseline model with homogeneous and with heterogeneous diversification. The coefficient of relative risk aversion γ is set at 7 and the standard deviation of the under-diversified stock portfolio is twice the one on the diversified portfolio.

	Participation rate	Conditional share
Data	48.9	59.6
Homogeneous diversification	76.5	79.7
Heterogeneous diversification	51.3	64.2

Table 2

Participation rates by age.

Table 2 reports the average participation rates over the life-cycle by 10 year age groups up to age 70 plus a single group for older ages. The top line refers to the US data from the 1998 SCF and is based on Guiso et al. (2001). The second and third line report respectively the values from the baseline model with homogeneous and with heterogeneous diversification. The coefficient of relative risk aversion γ is set at 7 and the standard deviation of the under-diversified stock portfolio is twice the one on the diversified portfolio.

	30-	30-40	40-50	50-60	60-70	70+
Data	34.3	51.8	58.3	61.4	47.1	32.4
Baseline						
Homogeneous diversification	39.4	84.7	90.6	91.5	85.1	72.8
Heterogeneous diversification	20.2	46.7	59.4	66.0	67.2	55.5

Table 3

Conditional shares by age.

Table 3 reports the average conditional stock share over the life-cycle by 10 year age groups up to age 70 plus a single group for older ages. The top line refers to the US data from the 1998 SCF and is based on Guiso et al. (2001). The second and third line report respectively the values from the baseline model with homogeneous and with heterogeneous diversification. The coefficient of relative risk aversion γ is set at 7 and the standard deviation of the under-diversified stock portfolio is twice the one on the diversified portfolio.

	30-	30-40	40-50	50-60	60-70	70+
Data	52.0	53.4	61.0	61.4	60.8	57.9
Baseline						
Homogeneous diversification	99.9	94.9	81.1	73.9	67.3	65.6
Heterogeneous diversification	81.1	75.8	70.5	65.0	51.4	33.5

Table 4

Average participation rates: changing risk aversion.

Table 4 reports average participation rates in the model for three different values of the coefficient of relative risk aversion γ, namely, from top to bottom: 9, 7 and 5. Each sub-panel refers to one value of the risk aversion coefficient. Results for the model with homogeneous diversification and with heterogenous diversification are reported in the first and second row of each sub-panel. In the models with heterogeneous diversification the standard deviation of the return on the under-diversified stock portfolio is kept at twice the value of the one of the return on the diversified portfolio as in the baseline case. Results are reported separately in the first column for the overall population and in the second and third column for high and low permanent income types.

	Average	High type	Low type
$\gamma = 9$			
Homogeneous diversification	80.5	88.7	71.4
Heterogeneous diversification	50.1	77.3	23.9
$\gamma = 7$			
Homogeneous diversification	76.5	85.5	67.7
Heterogeneous diversification	51.3	76.7	25.9
$\gamma = 5$			
Homogeneous diversification	69.4	79.0	59.7
Heterogeneous diversification	53.0	71.9	34.0

Table 5

Average conditional shares: changing risk aversion.

Table 5 reports average conditional stock shares in the model for three different values of the coefficient of relative risk aversion γ, namely, from top to bottom: 9, 7 and 5. Each sub-panel refers to one value of the risk aversion coefficient. Results for the model with homogeneous diversification and with heterogenous diversification are reported in the first and second row of each sub-panel. In the models with heterogeneous diversification the standard deviation of the return on the under-diversified stock portfolio is kept at twice the value of the one of the return on the diversified portfolio as in the baseline case. Results are reported separately in the first column for the overall population and in the second and third column for high and low permanent income types.

	Average	High type	Low type
$\gamma = 9$			
Homogeneous diversification	64.9	64.1	65.9
Heterogeneous diversification	46.3	49.7	35.4
$\gamma = 7$			
Homogeneous diversification	79.7	77.9	82.1
Heterogeneous diversification	64.2	66.7	56.6
$\gamma = 5$			
Homogeneous diversification	93.5	91.4	96.1
Heterogeneous diversification	81.1	85.3	72.2

Table 6

Average participation rates: changing stock portfolio variance.

Table 6 reports average participation rates for different values of the standard deviation of the under-diversified stock portfolio while keeping fixed the standard deviation of the diversified stock portfolio at 16 percent and the risk aversion coefficient γ at its baseline value of 7. In the first panel the results for the model with fully diversified stock portfolio only are reported. In the second panel I report results for three different values of the standard deviation of the return on the under-diversified portfolio, namely, from top to bottom: 1.5, 2 and 2.25 times the one on the return on the diversified portfolio. Results are reported separately in the first column for the overall population and in the second and third column for high and low permanent income types.

	Average	High type	Low type
Homogeneous diversification	80.5	88.7	71.4
Heterogeneous diversification			
$\sigma_h = 1.5 \times \sigma_l$	65.9	83.3	48.4
$\sigma_h = 2 \times \sigma_l$	51.3	76.7	25.9
$\sigma_h = 2.25 \times \sigma_l$	47.7	72.6	22.7

Table 7

Average conditional shares: changing stock portfolio variance.

Table 7 reports average conditional shares for different values of the standard deviation of the under-diversified stock portfolio while keeping fixed the standard deviation of the diversified stock portfolio at 16 percent and the risk aversion coefficient γ at its baseline value of 7. In the first panel the results for the model with fully diversified stock portfolio only are reported. In the second panel I report results for three different values of the standard deviation of the return on the under-diversified portfolio, namely, from top to bottom: 1.5, 2 and 2.25 times the one on the return on the diversified portfolio. Results are reported separately in the first column for the overall population and in the second and third column for high and low permanent income types.

	Average	High type	Low type
Homogeneous diversification	79.7	77.9	82.1
Heterogeneous diversification			
$\sigma_h = 1.5 \times \sigma_l$	63.3	69.3	52.6
$\sigma_h = 2 \times \sigma_l$	64.2	66.7	56.6
$\sigma_h = 2.25 \times \sigma_l$	65.1	66.7	59.8

Table 8

Wealth distribution at retirement.

Table 8 reports wealth shares in the year before retirement for selected wealth cells (top panel) and the wealth levels at the percentiles defining the boundaries of the given wealth cells (bottom panel). In each panel the top row refers to the model with a diversified stock fund only and the bottom row refers to the model with both a diversified and under-diversified stock fund.

Wealth shares				
Percentiles	0-40	0-60	80-100	90-100
Homogeneous diversification	4.5	15.0	63.1	43.0
Heterogeneous diversification	3.1	12.6	65.3	44.6
Wealth levels				
Percentiles	40	60	80	90
Homogeneous diversification	7.9	17.6	37.0	63.7
Heterogeneous diversification	5.4	16.8	36.7	63.5

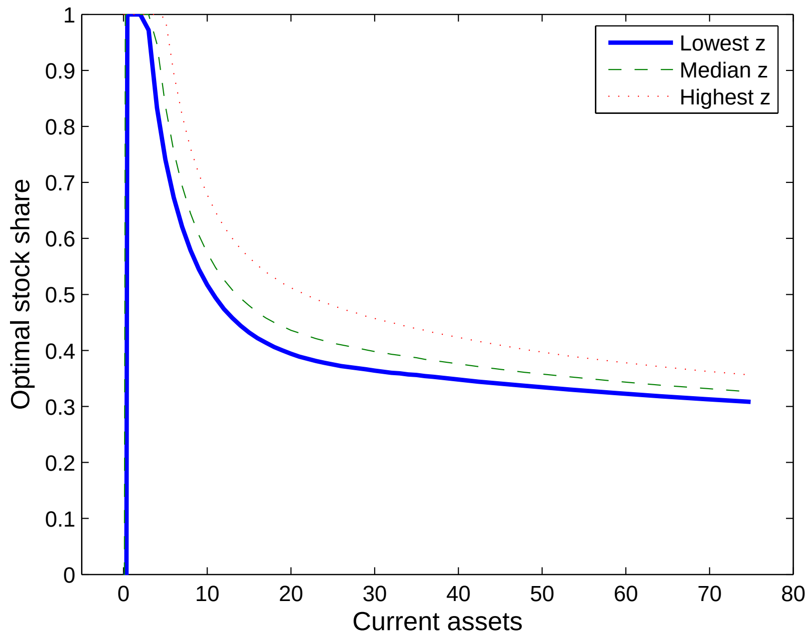

Figure 1: Stock share decision rules. Model with diversified stock investment.

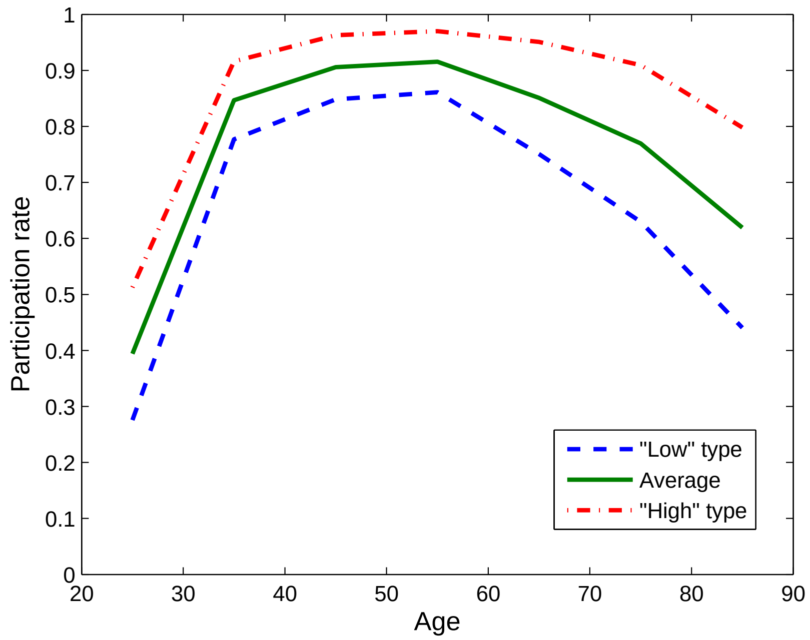

Figure 2: Life-cycle participation rates. Model with diversified stock investment.

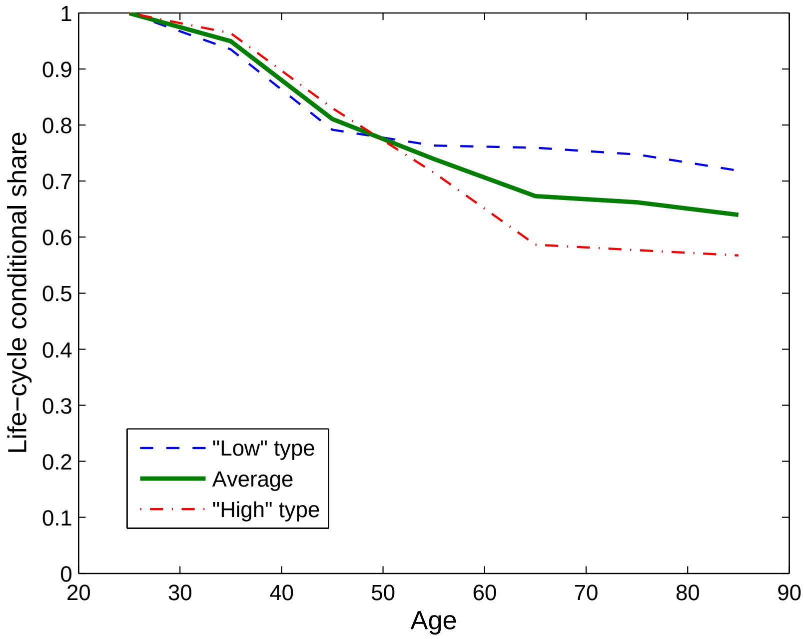

Figure 3: Life-cycle conditional stock shares. Model with diversified stock investment.

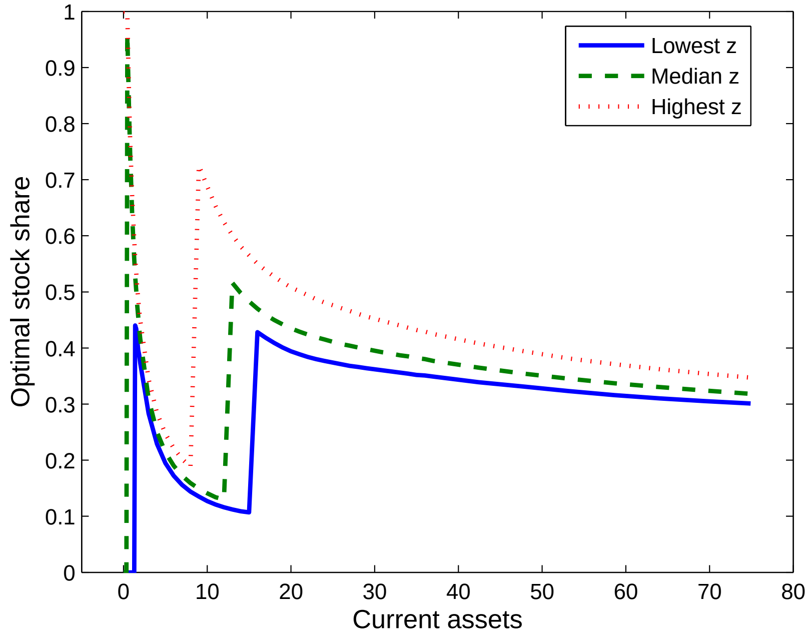

Figure 4: Stock share decision rules. Model with diversified and under-diversified stock investment.

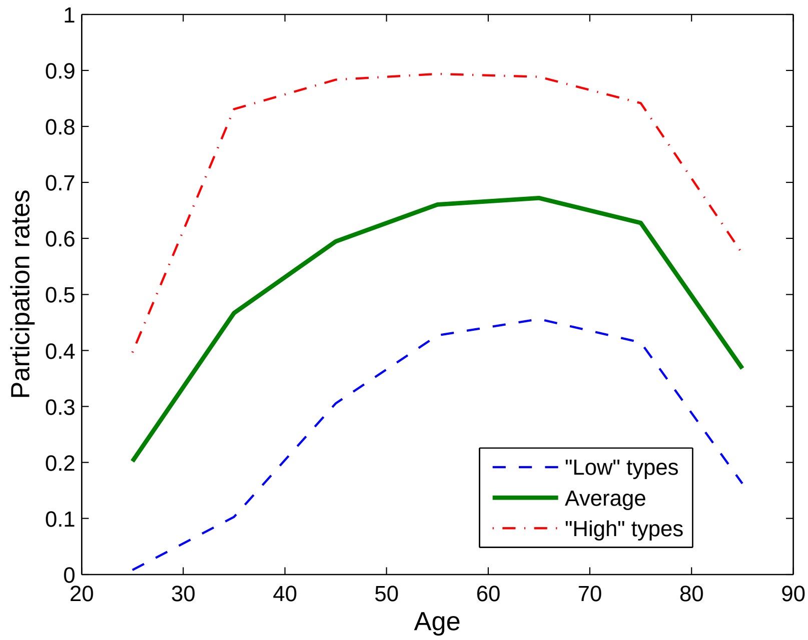

Figure 5: Life-cycle participation rates. Model with diversified and under-diversified stock investment.

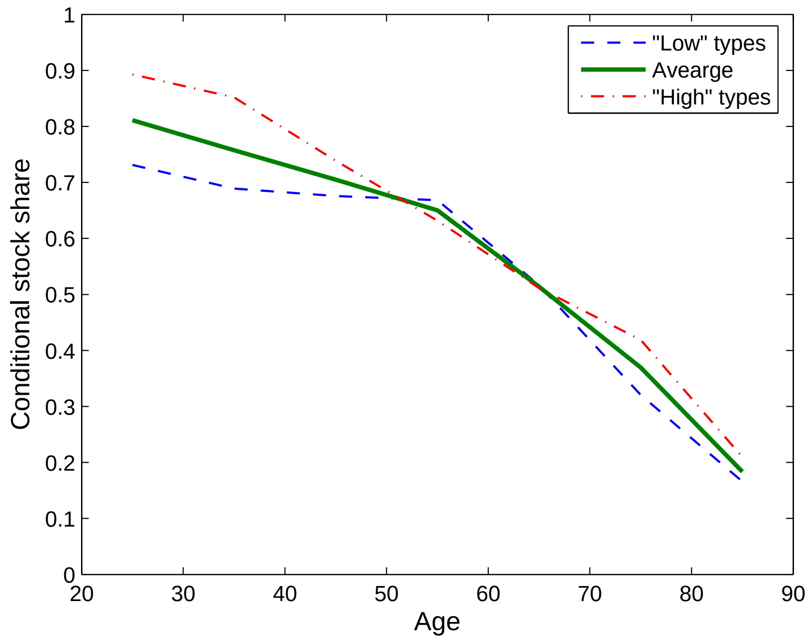

Figure 6: Life-cycle conditional stock shares. Model with diversified and under-diversified stock investment.